A PRECISION RECURSIVE ESTIMATE FOR EPHEMERIS REFINEMENT (PREFER)

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ABSTRACT

PREFER is a filter/smoother program for orbit determination which is used to refine the ephemerides produced by a batch least squares program (e.g., CELEST). PREFER requires, as input, a file containing the nominal satellite ephemerides and the state transition matrices as generated by CELEST. PREFER interpolates from this file at the times given on the Measurement Data file and processes the measurements in the Kalman filter to estimate the corrections to the nominal trajectory. The filter state also includes other parameters which have an effect upon the orbit determination (e.g., drag, perturbing gravitational accelerations, thrust, measurement biases and refraction parameters, etc.). Because PREFER is estimating the corrections to the nominal values, all partials are evaluated about the nominal trajectory and the filter is linear (not extended).

The measurement data types which PREFER can process include ground range, range difference and Doppler measurements, GPSPAC pseudorange and pseudodelta-range measurements, NAVPAC range difference measurements and altimeter measurements. A GPS Trajectory file supplies the ephemerides of the GPS satellites which are required to process the GPSPAC or NAVPAC measurements. A unique feature of the program is the capability to estimate hundreds of pass-disposable, measurement biases while using storage and computation for only a few biases.

After running the Kalman filter forward to the end of the Measurement Data file, PREFER performs optimal smoothing. A file created by the Kalman filter is read backward in time and the smoothed estimates are obtained by using the recursive formulation of Rauch-Tung-Striebal.

The combination of a Kalman filter and a smoother should result in greatly improved estimates of satellite ephemerides as compared to the batch estimation. Batch estimation is subject to errors because of errors in the dynamic models (e.g., gravitational). A filter/smoother which properly accounts for dynamic (state) noise should weight the data optimally and reduce the estimation errors. Smoothing will produce better estimates (in the middle of the data span) than just a forward filter because past and future data is used to estimate the state at each point in time (a filter uses only past data). Smoothing also tends to average out any dynamic modeling errors which remain.

PREFER's capability for improving orbit determination has been demonstrated on simulated data which contained significant modeling errors. The nominal trajectory had errors as large as 53 meters and the GPS trajectory file had peak errors of 12 meters. However, the PREFER smoother estimate was usually accurate to 3 meters with peak errors of 8 meters. Even during data gaps, the smoothed radial error was always less than 6 meters.

INTRODUCTION

A recursive filter/smoother orbit determination program has been developed to refine the ephemerides produced by a batch orbit determination program (e.g., CELEST, GEODYN). PREFER can handle a variety of ground and satellite-to-satellite tracking types as well as satellite altimetry. It has been tested on simulated data which contained significant modeling errors and the results clearly demonstrate the superiority of the program compared to batch estimation.

Input

The input to the program consists of four files and card input. A file containing the nominal (batch estimate) host satellite ephemerides and the 6 by 6 state transition matrix (from epoch osculating elements to current cartesian elements) is interpolated at the times given on the measurement data file. A GPS trajectory file supplies the ephemerides of the GPS satellites which are required to process the GPSPAC or NAVPAC measurements. A sun/moon file supplies the data which is used in the earth motion model (for ground based measurements). The card input to the program specifies run constants (e.g., time intervals) and *a priori* standard deviations, state noise spectral densities, time constants, etc.

Measurement Types

PREFER can process the following types of measurements.

- Ground Tracking

  Satellite to ground range
  Ground laser range
  Satellite to ground range difference
  Ground Doppler

- Satellite-to-Satellite

  GPS pseudo range and pseudo delta range
  NAVPAC range difference

- Altimetry

  Range to center of earth.

Provisions have been made for handling 50 ground stations and 24 GPS satellites but only 4 ground stations and 15 GPS satellites can be simultaneously observable. This restriction is imposed because of a limitation on the total number of states. Since station position errors, measurement biases, refraction parameters, GPS position errors and timing biases can all be estimated, the state vector could become unwieldy. PREFER has the capability to estimate all these parameters while using storage and computation for only those parameters which are simultaneously observable. This is discussed in later sections. Thus, the limitation is on the number of simultaneously observable stations and GPS satellites. As a practical matter, this limitation is not very restricting since it is unlikely that more than four ground stations

would see a low altitude satellite. Furthermore, simulations have shown that for the 24 satellite GPS system, no more than 15 GPS satellites would be observable to a low altitude satellite (without encountering severe refraction problems).

The altimetry measurements are assumed to have been preprocessed with a nominal geoid model so that they are treated as a range to the center of the earth.

Dynamics

A list of the dynamic parameters which PREFER can estimate is given below:

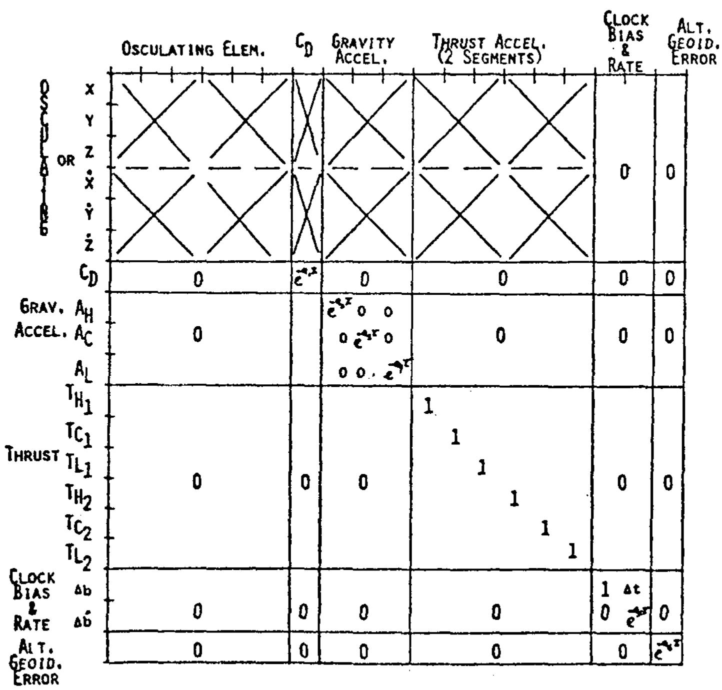
1 Satellite semimajor axis at epoch
2 Satellite eccentricity x *sin* (argument of perigee) at epoch
3 Satellite eccentricity x *cos* (argument of perigee) at epoch
4 Satellite inclination at epoch
5 Satellite mean anomaly plus argument of perigee at epoch
6 Satellite right ascension of ascending node at epoch
7 Satellite drag coefficient
8 Perturbing gravitational acceleration (vertical)
9 Perturbing gravitational acceleration (cross-track)
10 Perturbing gravitational acceleration (along-track)
11 Acceleration of 1st thrust segment (vertical)
12 Acceleration of 1st thrust segment (cross-track)
13 Acceleration of 1st thrust segment (along-track)
14 Acceleration of 2nd thrust segment (vertical)
15 Acceleration of 2nd thrust segment (cross-track)
16 Acceleration of 2nd thrust segment (along-track)
17 Host satellite clock timing error
18 Host satellite clock drift rate
19 Altimeter bias.

The first 6 are epoch osculating elements. The drag coefficient, perturbing gravitational accelerations, host clock drift rate and altimetry geoid error (bias) are all assumed to be independent, first order Markov processes. This may not be strictly true but it is a reasonable approximation. The thrust accelerations are assumed to be constant since the thrust durations will be relatively short.

The state transition matrix for the entire system of dynamic parameters and measurement related biases is:

$$\Phi = \begin{bmatrix} \phi_1 & 0 \\ 0 & I \end{bmatrix}$$

where ϕ_1 is:



The upper left 6 x 6 partition of ϕ_1 is an identity matrix when Φ is being used to perform the time update on the state vector. However, when individual measurements are being processed, the satellite position and velocity in cartesian coordinates at the measurement time must be known. The nominal position and velocity and the transition matrix from epoch osculating to cartesian elements are obtained by interpolation from the host trajectory file. The filter state (which includes the estimated correction to the epoch osculating elements) is multiplied by ϕ_1 to obtain the estimated correction to the nominal cartesian elements.

The upper right partition of ϕ_1 (i.e., the transition from C_d, gravitational accelerations and thrust to cartesian elements) is obtained as an iterated, second order Taylor series. Since the integration time interval will be relatively short (less than 120 seconds) and state noise is included in the formulation, a highly accurate integration method is not required.

The state noise covariance matrix (required by the filter) is obtained by Taylor series integration of the input spectral density matrix.

Kalman Filter

Measurements are processed in a Kalman filter to estimate the corrections to the nominal trajectory. All partial derivatives are evaluated about the nominal trajectory and thus the filter is linear (not extended).

Since the program was intended to process many thousands of measurements, the execution time would have been excessive if the Kalman equations were evaluated for each measurement. Therefore, the measurements are processed in small "mini-batches" (typically 120

seconds), during which time, the dynamics are assumed to be deterministic. Only when proceeding from the epoch of one mini-batch to the next is state noise included in the covariance equations. The term "mini-batch" is intended to indicate the lack of state noise rather than the method of processing since the estimation algorithm is actually the recursive U-D algorithm of Bierman [1].

A unique feature of PREFER is the capability to estimate hundreds of pass-disposable measurement-related biases while using storage and computation for only a few. As measurement data from new stations or GPS satellites is processed, the state vector and covariance matrix are augmented with the *a priori* information for the new measurement parameters. When the station or GPS satellites are no longer visible to the host satellite, the parameters are dropped from the state vector and covariance matrix. These parameters can be deleted from the filter state since they will no longer have an influence on the estimation of "common" parameters (dynamic and other measurement related biases). However, the deletion of parameters from the filter state does complicate smoothing since the lost information must be reconstructed later. This is discussed in another section.

It should be noted that these hundreds of measurement related parameters are probably not observable in a statistical sense, i.e., *a priori* information is required to make the covariance matrix full rank. These parameters are included in the filter state primarily to assure proper weighting of the measurement data.

Figure 1 is a flow chart of the FILTER subroutine. This routine is called once for each mini-batch of data. The flow chart shows the sequence of events required to perform the time update, write information on the disk for smoothing, process data with the U-D algorithm and delete parameters from the filter state.

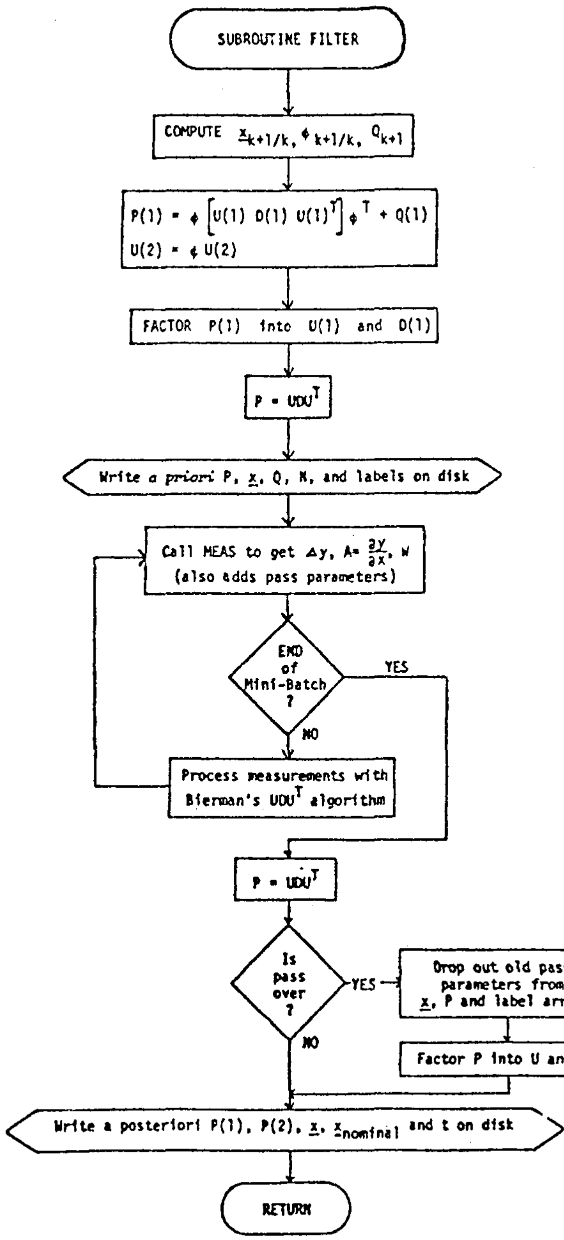


Figure 1 Filter Subroutine

Smoothing

Optimal smoothing is performed using the backward recursion developed by Rauch, Tung and Striebel [4]. The final estimate of the filter is used to initialize the smoother equations. The smoother gain matrix at time t_k is computed as:

$$G_k = \Phi_{k+1}^{-1}(I-Q_{k+1}P_{k+1/k}^{-1}) \quad .$$

Then the smoothed state vector and covariance are computed as:

$$\hat{\underline{x}}_{k/m} = \hat{\underline{x}}_{k/k} + G_k(\hat{\underline{x}}_{k+1/m}-\hat{\underline{x}}_{k+1/k})$$

$$P_{k/m} = P_{k/k} + G_k(P_{k+1/m}-P_{k+1/k})\ G_k^T$$

where the notation $\hat{\underline{x}}_{i/j}$ means the estimate $\underline{x}$ at time t_i based upon measurements up to time t_j . In other words, $\hat{\underline{x}}_{k+1/k}$ is the *a priori* estimate at time t_{k+1}, $\hat{\underline{x}}_{k/k}$ is the *a posteriori* estimate at time t_k and $\hat{x}_{k/m}$ is the smoothed estimate at time t_k (t_m is the last data point).

Notice that the gain matrix G_k has the following structure:

$$G_k = \begin{bmatrix} G(1) & G(2) \\ & \\ 0 & I \end{bmatrix}$$

where the partitioning indicated separates the dynamic parameters from the biases. Since the number of biases may be several times greater than the number of dynamic parameters, the multiplications by 0 or 1 are avoided in the coding.

Although Kalman filter formulations based upon covariance matrices are more prone to numerical problems than the factored filters, numerical problems are not so severe in the smoother. The smoother equations are only evaluated once per mini-batch rather than for each measurement. Furthermore, the equations for the smoothed $\underline{x}$ and P are uncoupled since the gain matrix only depends upon variables from the filter. Thus, errors in the smoothed P have no effect upon $\underline{x}$.

Disposable Pass Parameters in Smoothing

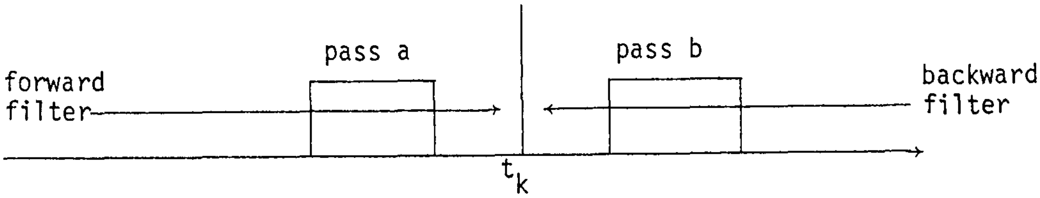
It is fairly well known that measurement bias parameters need only be included in the filter state during periods when data of the appropriate type is actually being processed. Outside the data interval, the solution for the pass parameters has no effect upon the solution for the common parameters.

We are not aware of any published reference which demonstrates that the "disposable parameter" approach is also valid for smoothing. Therefore, this section shows that the approach is valid and demonstrates how it is implemented for the present problem. The following derivation is basically the same as that given by Tanenbaum.[1]

Fraser and Potter [2] showed that the optimum smoother could also be derived as the linear combination of a forward filter which includes *a priori* information and a backward filter which does not include *a priori*. The results obtained from such a filter will be identical to those obtained by the RTS algorithm.

Consider the case shown in the figure where the forward filter has processed data from pass a but not b while the backward filter has processed data from b but not a .

[1]Tanenbaum, M., private communication, NSWC/Dahlgren, December 1977.



The filter states and covariances at time t are:

Forward	Backward
$\underline{x}_k = \begin{bmatrix} \underline{x}_c \\ \underline{x}_a \\ 0 \end{bmatrix}$	$\underline{x}'_k = \begin{bmatrix} \underline{x}'_c \\ 0 \\ \underline{x}'_b \end{bmatrix}$
$\begin{bmatrix} P_{cc} & P_{ca} & 0 \\ P_{ac} & P_{aa} & 0 \\ 0 & 0 & \infty \end{bmatrix}$	$\begin{bmatrix} P'_{cc} & 0 & P'_{cb} \\ 0 & \infty & 0 \\ P'_{bc} & 0 & P'_{bb} \end{bmatrix}$

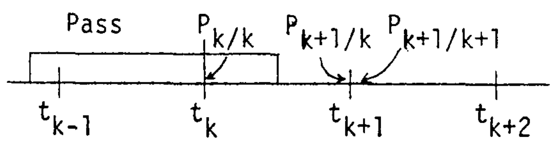
where subscript c denotes common parameters. Notice that the *a priori* information for the pass b parameters of the foward filter is treated as if it is a measurement which does not actually enter the forward filter until the pass is begun. It can also be shown (with some difficulty) that similar results are obtained by allowing it to enter the filter at the initial time. The smoothed covariance is obtained as a minimum variance combination of the two estimates. Since the errors in the two estimates are uncorrelated, the smoothed covariance is simply the inverse of the sum of the two information matrices*

*Operations on matrices containing ∞ must be done with great care. The result can, however, be derived more rigorously.

$$P_{k/m} = (P_k^{-1} + P_k'^{-1})^{-1}$$

$$= \left[\begin{array}{c|c|c} (P_{cc}^{-1}+P_{cc}'^{-1})^{-1} & P'_{cc}(P_{cc}+P'_{cc})^{-1}\ P_{ca} & P_{cc}(P_{cc}+P'_{cc})^{-1}\ P'_{cb} \\ & P_{aa}-P_{ac}(P_{cc}+P'_{cc})^{-1}\ P_{ca} & P_{ac}(P_{cc}+P'_{cc})^{-1}\ P'_{cb} \\ & & P'_{bb}-P'_{bc}(P_{cc}+P'_{cc})^{-1}\ P'_{cb} \end{array}\right]$$

Notice that the solution for the common parameters does not depend upon the pass parameters. Furthermore, the solution for pass a does not depend upon the pass b parameters (and vice versa). This verifies that it is not necessary to carry the pass parameters outside of the pass. However, we must also verify that the pass parameters can be "reconstructed" in the RTS formulation of the smoother.



Consider the case shown in the figure. Assume that the smoothed values for t_k are to be computed. $P_{k/k}$ and $P_{k+1/k}$ from the forward filter have the same dimension but $P_{k+1/k+1}$ does <u>not</u> include the pass parameters. Obviously, the smooth covariance, $P_{k+1/m}$, is the same dimension as $P_{k+1/k+1}$. In the RTS equations, the difference $P_{k+1/m} - P_{k+1/k}$ must be computed but these two arrays are of different dimensions. Therefore, we examine whether the missing terms of ΔP can be reconstructed. Using the results from the forward-backward smoother, we find that:

$$P_{k+1/m} - P_{k+1/k} = \left[\begin{array}{c|c|c} \Delta P_{cc} & \Delta P_{cc}(P_{cc}^{-1}P_{ca}) & -\Delta P_{cc}(P_{cc}^{-1}P'_{cb}) \\ & (P_{ac}P_{cc}^{-1})\Delta P_{cc}(P_{cc}^{-1}P_{ca}) & -(P_{ac}P_{cc}^{-1})\Delta P_{cc}(P_{cc}^{-1}P'_{cb}) \\ & & -\infty \end{array}\right]$$

where $\Delta P_{cc} = -P_{cc}(P_{cc}+P'_{cc})^{-1} P_{cc}$ is simply computed as the upper left partition of $P_{k+1/m} - P_{k+1/k}$.

When written in this form, it is obvious that ΔP_{k+1} is singular. This also shows that the "missing" terms of ΔP can be reconstructed by pre- or post-multiplying by the factor $P_{ac} P_{cc}^{-1}$ obtained from $P_{k+1/k}$. The rational for discarding pass parameters <u>after</u> writing the filter *a priori* to the disk should now be obvious.

By a similar procedure, we can also demonstrate that the pass parameter portion of $\underline{x}_{k+1/m} - \underline{x}_{k+1/k}$ can be reconstructed as

$$\Delta\underline{x}_{k+1} = \begin{bmatrix} \Delta\underline{x}_{cc} \\ \\ (P_{ac}P_{cc}^{-1})\ \Delta\underline{x}_{cc} \end{bmatrix}_{k+1} .$$

The equation for the gain matrix requires that $P_{k+1/k}$ be inverted. It can be easily shown [3] that the same results for the smoothed $\underline{x}$ and P will be obtained whether or not the pass parameters are included in the gain computation. Thus, the final RTS equations used when reconstructing pass parameters are:

$$\underline{x}_{k/m} = \underline{x}_{k/k} + G'\ \Delta\underline{x}_{cc}$$

$$P_{k/m} = P_{k/k} + G'\ \Delta P_{cc}\ G'^{T}$$

where

$$G' = \begin{bmatrix} \phi_{cc}^{-1}\ (I - Q_{cc}P_{cc}^{-1}) \\ \\ P_{ac}P_{cc}^{-1} \end{bmatrix}_k$$

Examples

Two examples using simulated data are given to demonstrate the improved performance of PREFER. The first is relatively trivial in that no modeling errors were included. The test was made simply to evaluate the program response to an initial condition error. Table 1 summarizes the test case and Figure 2 displays the results. The filter position error was initially 20 meters. During the first data pass, the error was reduced to 7 meters but during the subsequent data gap, the error rose to 38 meters. After the first orbit, the filter error remained below 1 meter. However, the smoother position error was less than 1.2 meters for the entire run. The smoother error is largest at epoch because the 1 sigma *a priori* error is weighted into the solution.

The second example is a more rigorous test of the program. It includes some additional data types and also has significant force modeling errors. Table 2 summarizes the input and Figure 3 displays the results.

The filter estimate has peak errors of 63 meters (mostly cross-track) while the maximum error in the smoother estimate is 11.2 meters (mostly radial) at the epoch. The peak error in the filter estimate occurs at 30 to 40 minutes which corresponds to a minimum error in the nominal trajectory. Apparently the filter had an erroneous estimate of the gravitational accelerations at the time that a data gap occurred.

ORBIT	-	350-420 KM ALTITUDE, $e = .005$, 96.9° INCLINATION, 180 MINUTES (2 REVOLUTIONS)
MODEL ERRORS	-	NONE (NOMINAL TRAJECTORY IS PERFECT)
TRACKING DATA	-	7 GROUND STATIONS, RANGE DATA ONLY, NO MEASUREMENT NOISE BUT DATA IS GIVEN A WEIGHT OF 1 METER
ADJUSTED PARAMETERS	-	ORBITAL ELEMENTS, MEASUREMENT BIAS AND REFRACTION PARAMETERS, STATION POSITION ERRORS
INITIAL CONDITIONS	-	FILTER ESTIMATE OF SEMI-MAJOR AXIS AT EPOCH IS PERTURBED BY 20 METERS (1σ)

A PRIORI STANDARD DEVIATIONS

SEMI-MAJOR AXIS	-	20 M
$e \, SIN \, \omega$	-	.00001 RADIAN
$e \, COS \, \omega$	-	.00001 RADIAN
INCLINATION	-	.00001 RADIAN
$\ell + \omega$	-	.00001 RADIAN
Ω	-	.00001 RADIAN
STATION BIAS	-	1 M
STATION REFRACTION	-	0.5 M
STATION POSITION	-	5 M (EACH COMPONENT)

STATE NOISE SPECTRAL DENSITY

X, Y, Z	-	.03 $M/SEC^{1/2}$
$\dot{X}$, $\dot{Y}$, $\dot{Z}$	-	$.3 \times 10^{-4}$ $M/SEC^{3/2}$

Table 1 Summary of Test Case Number 1

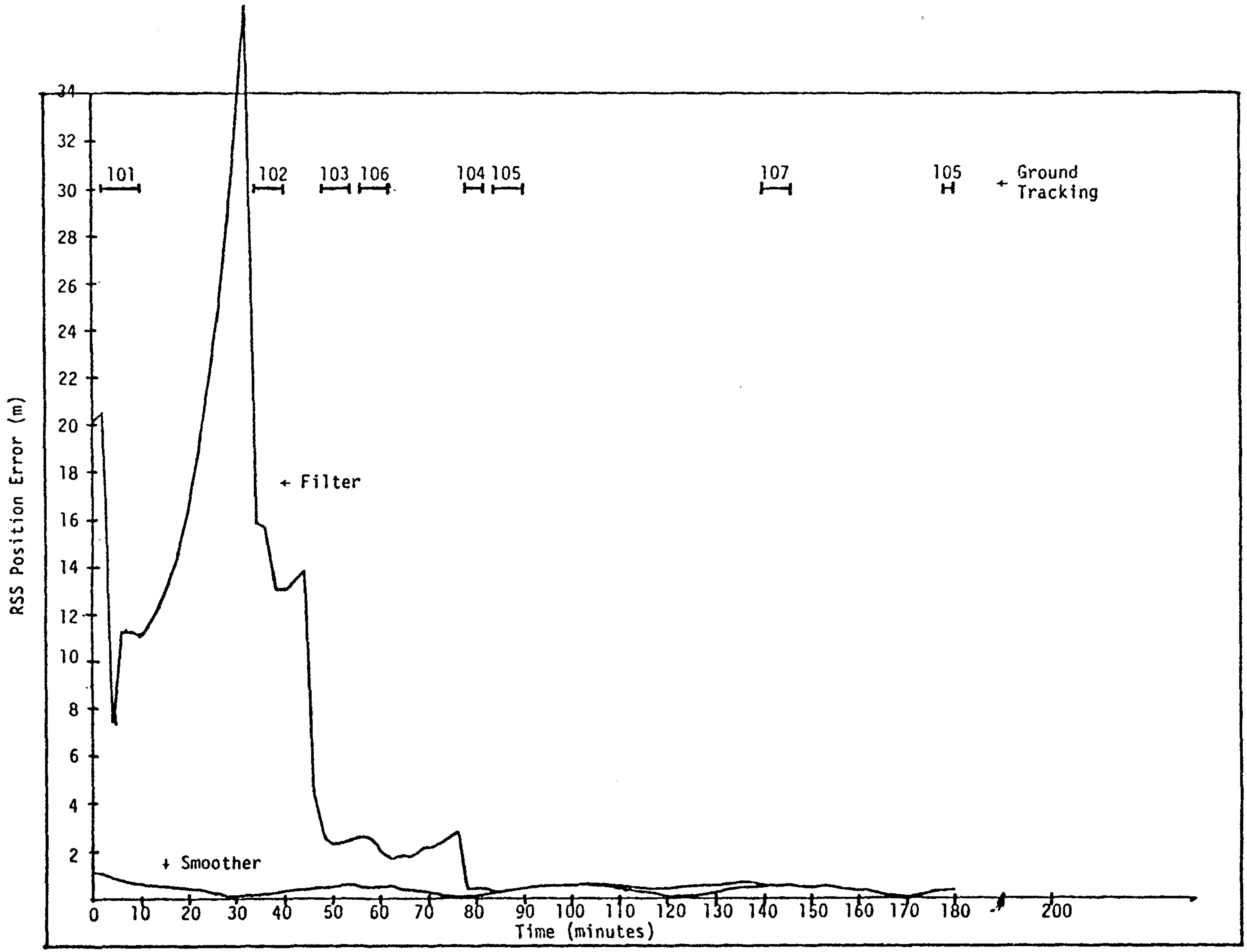


Figure 2 Error in Estimated Position for Example 1

ORBIT	-	165-264 KM ALTITUDE, e = .0075, 96.4° INCLINATION, 192 MINUTES (2 REVOLUTIONS)
MODEL ERRORS	-	MEASUREMENT DATA GENERATED USING A 25,25 GRAVITY FIELD. NOMINAL TRAJECTORY WAS OBTAINED BY LEAST SQUARES FITTING THE TRUE TRAJECTORY USING A 8,8 GRAVITY FIELD. THE RESULTING POSITION ERRORS ARE LESS THAN 53 METERS. ALSO, SINUSOIDAL ERRORS WERE ADDED TO THE POSITIONS ON THE GPS TRAJECTORY FILE. THE STANDARD DEVIATIONS FOR THE PEAK ERRORS WERE: 10 METERS ALONG-TRACK, 6 METERS CROSS-TRACK AND 2 METERS RADIALLY.
TRACKING DATA	-	16 GROUND STATIONS: ALL HAVE RANGE DATA BUT TWO ALSO HAVE RANGE DIFFERENCE AND ANOTHER TWO HAVE DOPPLER DATA. DATA IS NOISELESS BUT IS GIVEN WEIGHTS OF 1 METER (RANGE), 6 CM (RANGE DIFFERENCE) AND 0.2×10^{-10} (DOPPLER). 6 GPS SATELLITES (PSEUDO RANGE AND DELTA-RANGE). DATA HAS MEASUREMENT NOISE OF 1.5 METERS (PSEUDO RANGE) AND 2 CM (PSEUDO DELTA-RANGE). DATA IS WEIGHTED ACCORDINGLY.
ADJUSTED PARAMETERS	-	C_D, GRAVITATIONAL ACCELERATION, HOST CLOCK ERRORS, STATION MEASUREMENT BIASES AND REFRACTION, STATION POSITIONS, GPS POSITIONS AND TIMING.

Table 2 Summary of Test Case Number 2

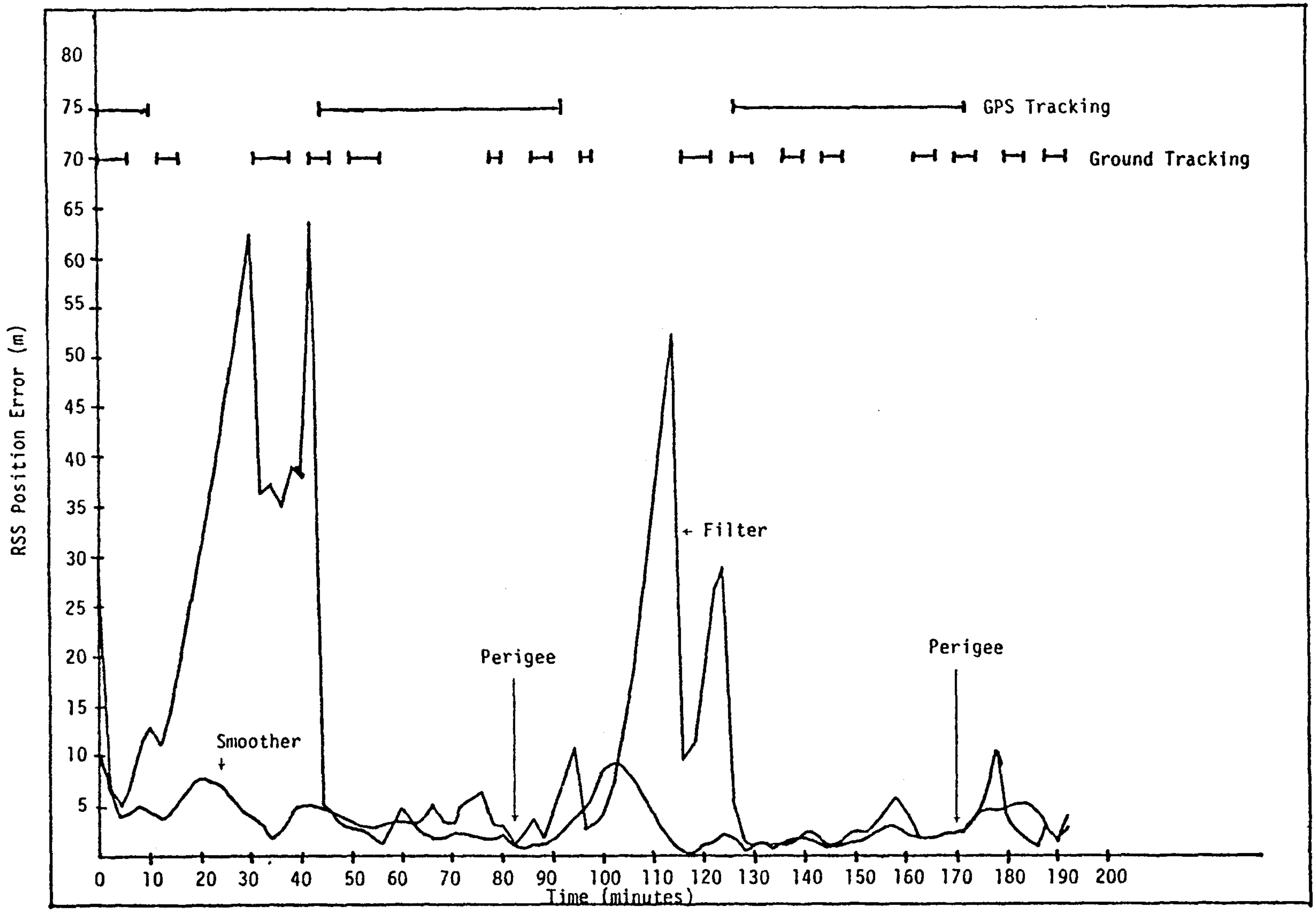


Figure 3 Error in Estimated Position for Example 2

Thus, the error quickly increased until more tracking was obtained. However, the filter covariance matrix during the data gaps was also large so that the smoother could correctly weight the filter estimates.

Notice that both the filter and smoother estimates are quite accurate during the periods when GPS tracking is available. During these periods, the smoother estimation error was generally less than three meters and the radial component was accurate to within 1.5 meters. Even during the data gaps, the smoother radial error did not exceed 6 meters (except at the epoch). This large error occurred at 102 minutes from epoch and the nominal trajectory at this time had a 50 meter cross-track error.

It should be noted that no great attempt was made to "fine tune" the input parameters for this example. Presumably the errors could be reduced further by the appropriate choice of state noise variances, time constants, etc.

Summary

The results of the various tests on simulated data demonstrate that PREFER has great potential for improving orbit determination of low altitude satellites.

References

1. Bierman, G. J., Factorization Methods for Discrete Sequential Estimation, Academic Press, 1977.

2. Fraser, D. C. and J. E. Potter, "The Optimum Linear Smoother as a Combination of Two Optimum Linear Filters", IEEE Transactions on Automatic Control, August 1969.

3. Gibbs, F. P., "Precision Recursive Estimator for Ephemeris Refinement (PREFER) Mathematical Description", Business and Technological Systems, Inc., TR-78-66, September 1978.

4. Rauch, H. E., F. Tung, and C. T. Striebel, "Maximum Likelihood Estimates of Linear Dynamic Systems", AIAA Journal, Vol. 3, No. 8, August 1965, pp. 1445-1450.